# Large population limits of Markov processes on random networks

Marvin Lücke, Jobst Heitzig, Péter Koltai, Nora Molkenthin, Sefanie Winkelmann

This talk accompanies the preprint [1]. We consider time-continuous Markovian discrete-state dynamics on random networks of interacting agents. Models of this kind can be used for example in epidemiology, where each node represents a person that can be susceptible, infectious or removed, edges represent contacts, and the dynamics describes how nodes switch between these states based on their contacts.

In this work we study the large population limits of such models. The dynamics are projected onto low-dimensional collective variables given by the shares of each discrete state in the system, or in certain subsystems. In the example from epidemiology, these shares describe the percentages of susceptible, infectious and removed nodes, either for the whole population or for subsets of the population.

We prove general conditions for the convergence of the collective variable dynamics to a mean-field ordinary differential equation (MFE), which describes the time evolution of the aforementioned shares. Furthermore, we discuss the convergence to this mean-field limit for the example of a continuous-time noisy version of the famous "voter model" on Erdős-Rényi random graphs, on the stochastic block model (see Figures 1 and 2), as well as on random regular graphs. For each of these types of interaction networks, we specify the convergence conditions in dependency on the corresponding model parameters.

Finally, we will discuss some statistical approaches for learning collective variables and their evolution from dynamical data, e.g., system trajectories.

## References

[1] Marvin Lücke, Jobst Heitzig, Péter Koltai, Nora Molkenthin, and Stefanie Winkelmann. Large population limits of markov processes on random networks, 2022. URL: https://arxiv.org/abs/2210.02934.

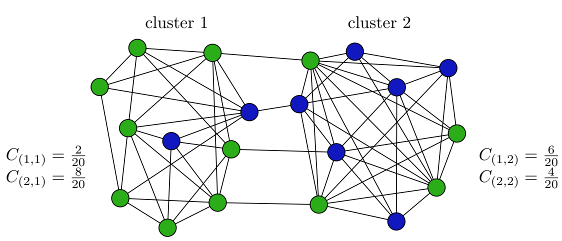


Figure 1: Example graph $G$ of size $N = 20$, sampled from a stochastic block model with two clusters. Each node has one of two states, state 1 is indicated by blue color and state 2 by green color. The collective variable $C(x)$ measures the shares of the two states in each cluster.

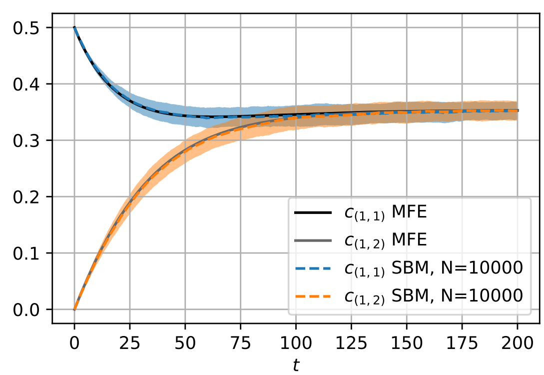


Figure 2: Mean (dotted line) ± one standard deviation (shaded area) of the voter model on a stochastic block model (SBM) with two equal size blocks, in comparison with the mean-field solution (MFE). Initially, all nodes in block 1 have state 1 and in block 2 state 2.